Theoretical corrections for the superallowed β decays of the proton-rich nuclei: ^{42}Ti, ^{46}Cr, ^{50}Fe, and ^{54}Ni

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In January 2015, Molina et al.[1] reported a measurement of the half-lives and Gamow-Teller branching ratios of the β decays of ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni, convincingly demonstrating that these nuclei are accessible and potentially amenable to more precise measurements. Therefore, it is appropriate that the radiative and isospin-symmetry-breaking corrections be likewise computed so that these transitions can be added to the data base of superallowed Fermi transitions [2] that are used in the determination of weak-interaction parameters, such as the CKM matrix element V_{ud}.

A β transition is characterized by its ft value, where f is the statistical rate function and t is its partial half-life. To the ft value, two theoretical corrections are applied to produce a corrected $\mathcal{F}t$ value, which is defined as

$$\begin{aligned} \mathcal{F}t &= ft\,(1+\delta_R)(1-\delta_C) \\ &= ft\,(1+\delta_R')(1-\delta_C+\delta_{NS}) \end{aligned} \tag{1}$$

Here δ_R is the nucleus-dependent part of the radiative correction, and δ_C is the isospin-symmetry-breaking correction. It is convenient to subdivide δ_R further as $\delta_R = \delta_R' + \delta_{NS}$ and, since the quantities are small, rearrange the equation to the form displayed in the second line of Eq. (1), which is correct to first order in these corrections. This rearrangement places the nuclear-structure-dependent corrections together in the combination δ_C - δ_{NS}.

The calculated radiative corrections, δ_R' and δ_{NS}, for the proton-rich nuclei considered here are given in Table I. The correction δ_R' comprises the bremsstrahlung and low-energy part of the γW – box graphs and is a standard QED calculation that depends only on the electron's energy and the charge Z of the daughter nucleus. The second component δ_{NS} recognizes that the γW – box graph includes situations in which the γ-nucleon interaction in the nucleus does not involve the same nucleon as that participating in the W - nucleon interaction. When this happens, two distinct nucleons are actively involved and a

Table I. Calculated radiative corrections $\boldsymbol{\delta_R'}$ and $\boldsymbol{\delta_{NS}}$, and isospin-symmetry-breaking corrections δ_{C1} and δ_{C2}in percent units. The combination δ_C - δ_{NS} brings together all the corrections that are nuclear-structure dependent. From Ref. [3].

Parent nucleus	δ_R'	δ_{NS}	δ_{C1}	δ_{C2}	δ_C - δ_{NS}
^{42}Ti	1.427	-0.235(20)	0.105(20)	0.855(60)	1.195(66)
^{46}Cr	1.420	-0.175(20)	0.045(20)	0.715(85)	0.935(90)
^{50}Fe	1.439	-0.155(20)	0.025(20)	0.635(45)	0.815(53)
^{54}Ni	1.430	-0.165(20)	0.065(30)	0.725(60)	0.955(70)

detailed shell-model calculation is required to evaluate δ_{NS}. More details on how the calculation is performed is given in Ref. [4].

For calculational convenience, the isospin-symmetry-breaking correction δ_C is separated into two components

$$\delta_C = \delta_{C1} + \delta_{C2} \tag{2}$$

The idea is that δ_{C1} follows from a tractable shell-model calculation (usually with a model space of one major oscillator shell) in which specific charge-dependent terms are added to the effective charge-independent interaction. Unfortunately, this procedure does not permit all the ramifications of mixing via the Coulomb force to be included. The correction δ_{C2} models the impact of Coulomb mixing as a change in the proton radial function. In the β-decay matrix element there is an overlap between the radial functions of the proton and the neutron that participate in the transition, and it is the reduction from unity of the overlap integral that leads to the correction δ_{C2}. Details of how δ_{C1} and δ_{C2} are computed are given in Ref. [4] and the results for the proton-rich nuclei of interest here are given in Table I.

With these theoretical corrections now computed, the proton-rich nuclei ^{46}Cr, ^{50}Fe and ^{54}Ni can be added to the data base [2] of superallowed Fermi transitions. Results for ^{42}Ti were already included in the data base, but have been updated here to incorporate recent results pertaining to its charge radius.

[1] F. Molina *et al.*, Phys. Rev. C **91**, 014301 (2015).
[2] J.C. Hardy and I.S. Towner, Phys. Rev. C **91**, 025501 (2015).
[3] I.S. Towner and J.C. Hardy, Phys. Rev. C **92**, 055505 (2015).
[4] I.S. Towner and J.C. Hardy, Phys. Rev. C **77**, 025501 (2008).